



Lieven
Penninck

Angular spectrum representation of vector fields for supporting metasurface analysis and design

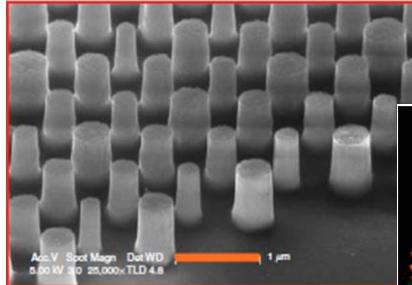
Peter Muys and Lieven Penninck

SPIE Optics + Photonics 2022, San Diego CA, USA

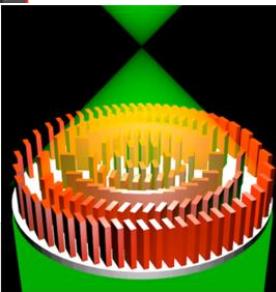


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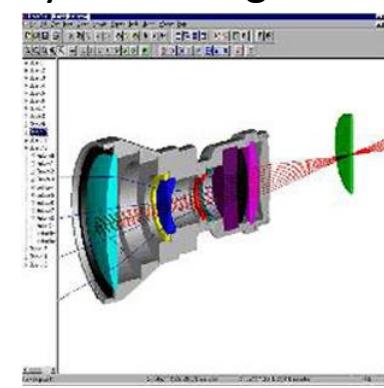
Nano-scale design



Component design



System Integration



Planopsim's mission

Planopsim supplies R&D tools to engineers & scientists that allow to unlock the maximum benefit of flat optics in a user-friendly way.

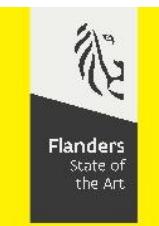


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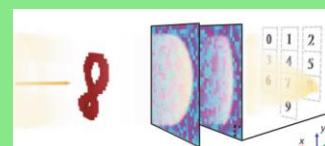
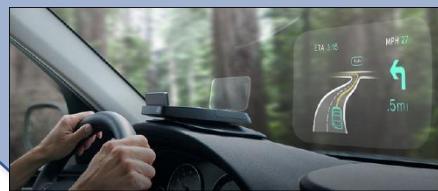
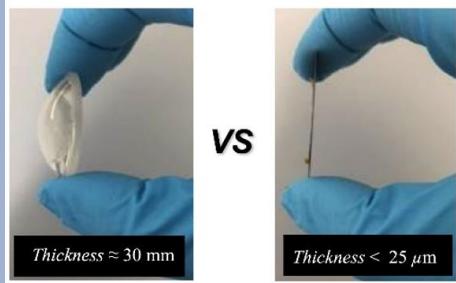


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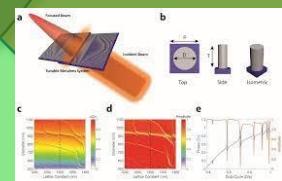
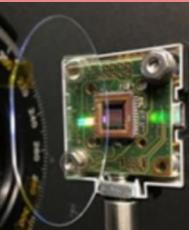
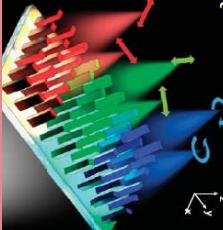
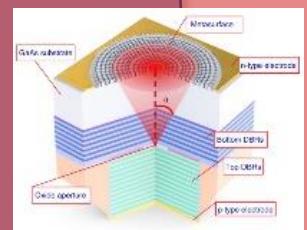
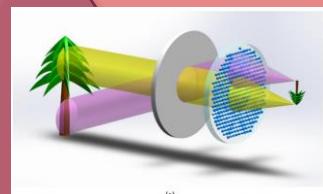
**AGENTSCHAP
INNOVEREN &
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Why use meta-surfaces?

Miniaturization

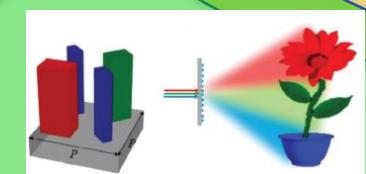
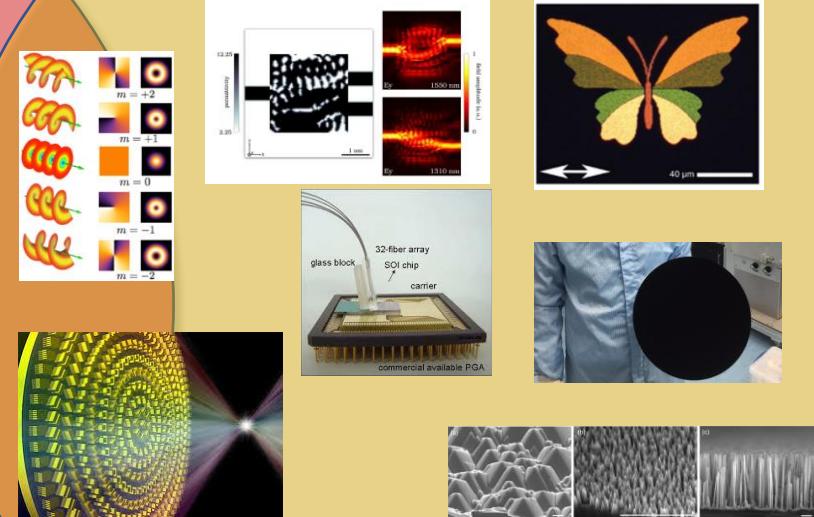


Simplification



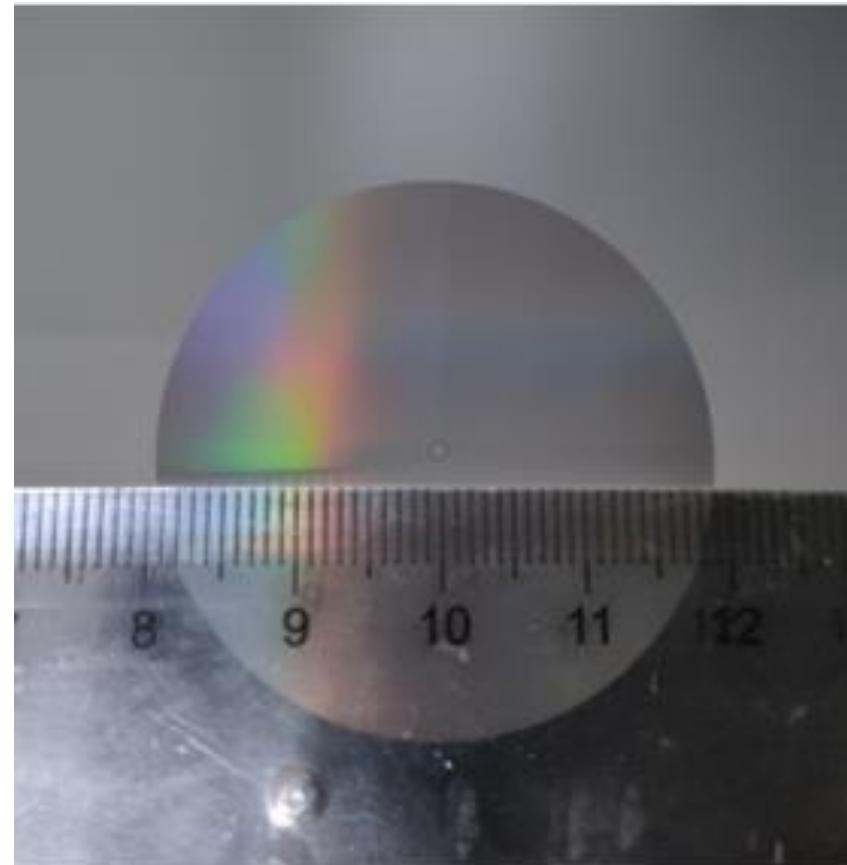
Invention

Functionalization



Getting to large meta-lenses

- ❖ Large = hard
 - Fabrication
 - Design methods
- ❖ Challenges in designing large metalenses
 - Multi-scale calculations needed
 - Memory limitations
 - Storage of 4cm diameter metlens : $20 \cdot 10^9$ element = 299Gb
- ❖ Design methodology
 - Combining different simulation scales
 - Meta-material informed system design
 - Modelling, quantifying and mitigating imperfections
- ❖ Fabrication:
 - Fabrication errors
 - Uniformity

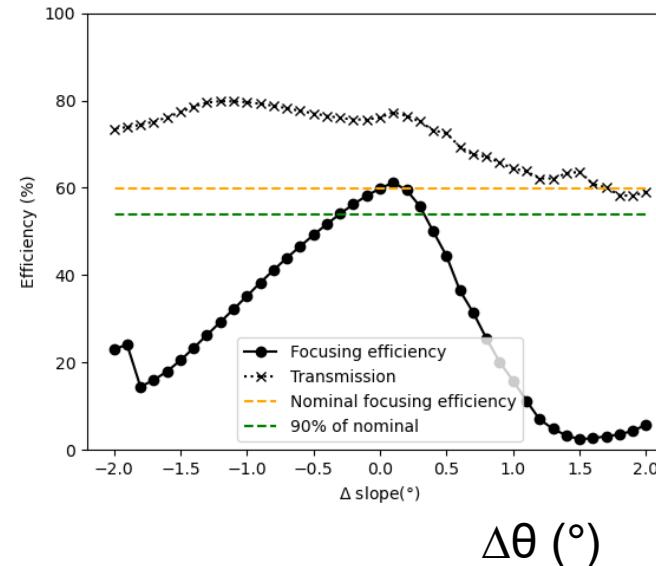


40mm diameter metlens

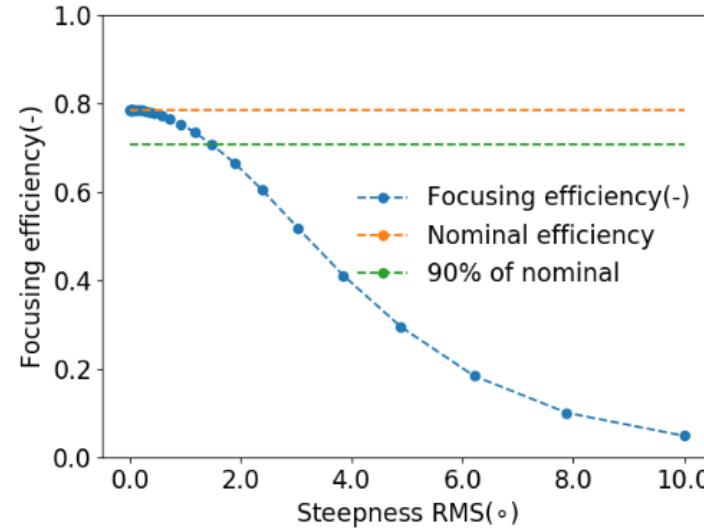
Comparison systematic to random error

- ❖ Previous work
- ❖ **Systematic errors** have a **stronger impact than random errors** on focussing efficiency
- ❖ Monte carlo results for meta-lens for 532nm

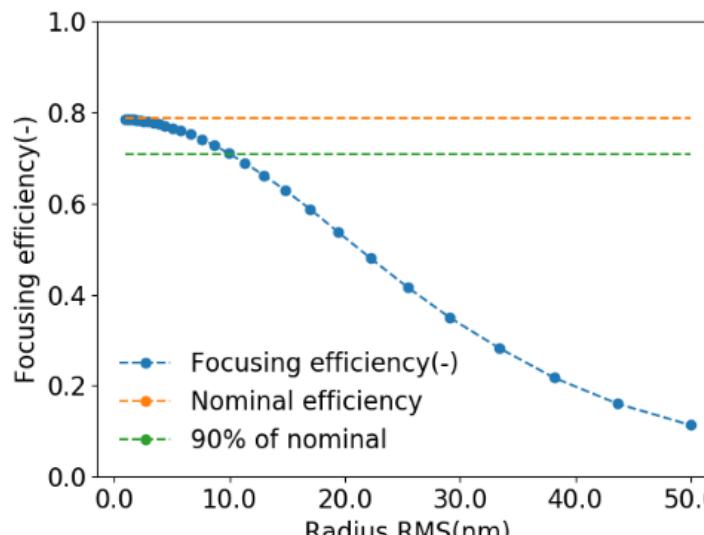
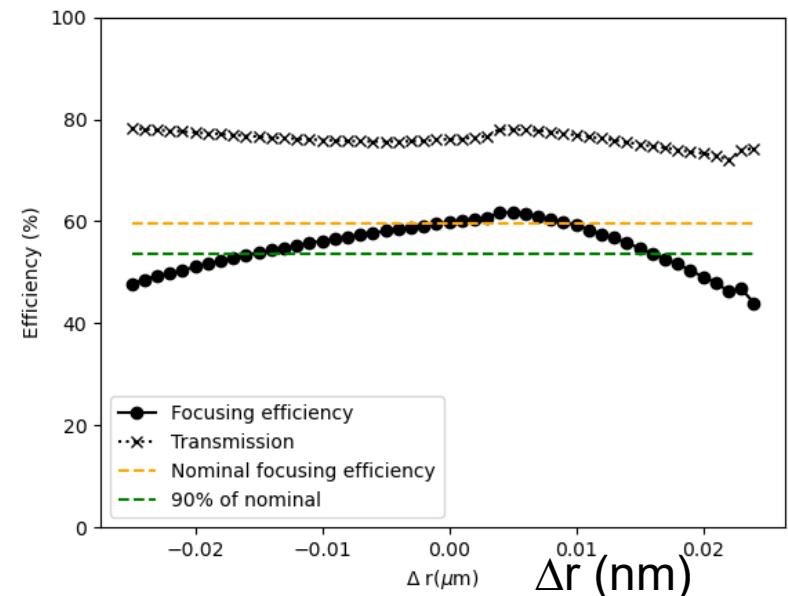
Systematic errors



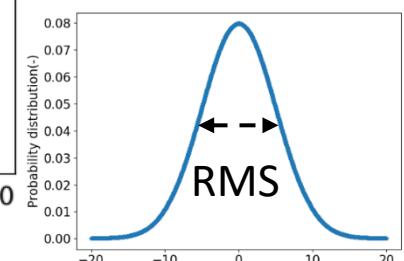
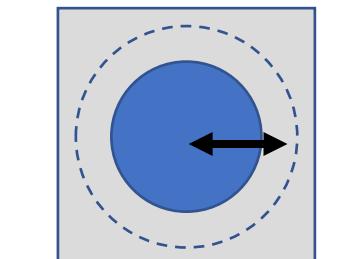
Random errors (Monte Carlo)



Steepness angle



radius modified

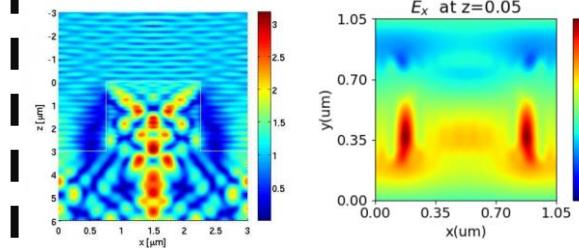


Multi-scale simulation

System Integration

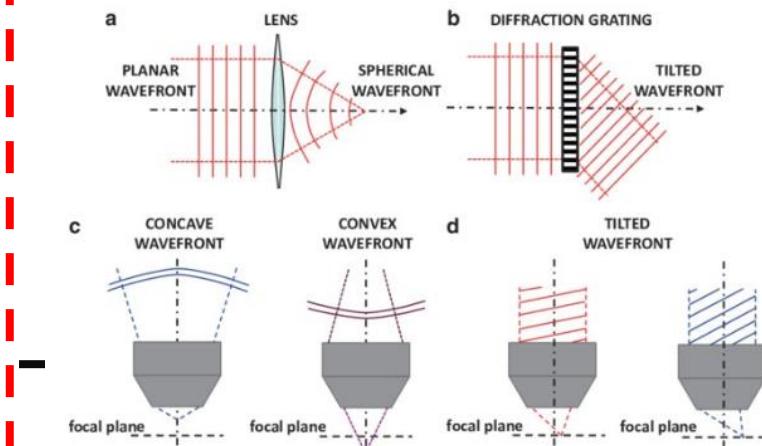
Individual components

Full wave solvers



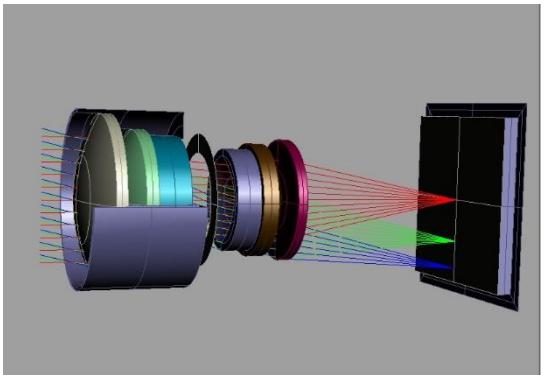
Nano-structure

Propagation optics



Maxwell's equations

Waves



Ray tracing

Geometric rays

Approaches to physical optics

❖ Richards-Wolfs integral

➢ 3FFT + brute force integration

$$\mathbf{E}(x, y, z) = -\frac{i\alpha}{\lambda} \iint_{\Omega} \underline{\mathbf{A}(k_x, k_y)} \exp(-i\mathbf{k} \cdot \mathbf{r}) d\Omega$$

❖ Angular spectrum method

➢ Vectorial

➢ 3 FFT + 3 IFFT

$$E(x, y, z) = \iint \underline{A(k_x, k_y; z=0)} \exp \left[-i(k_x x + k_y y + k_z z) \right] dk_x dk_y$$

❖ Scalar angular spectrum method

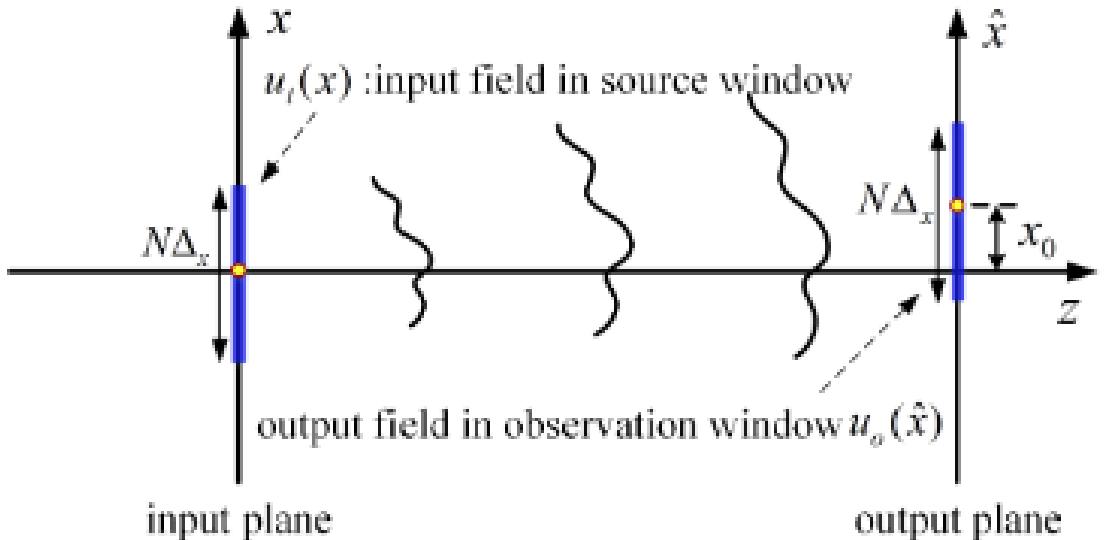
➢ 1FFT + 1 IFFT

❖ Fresnel integral

➢ Paraxial approximation

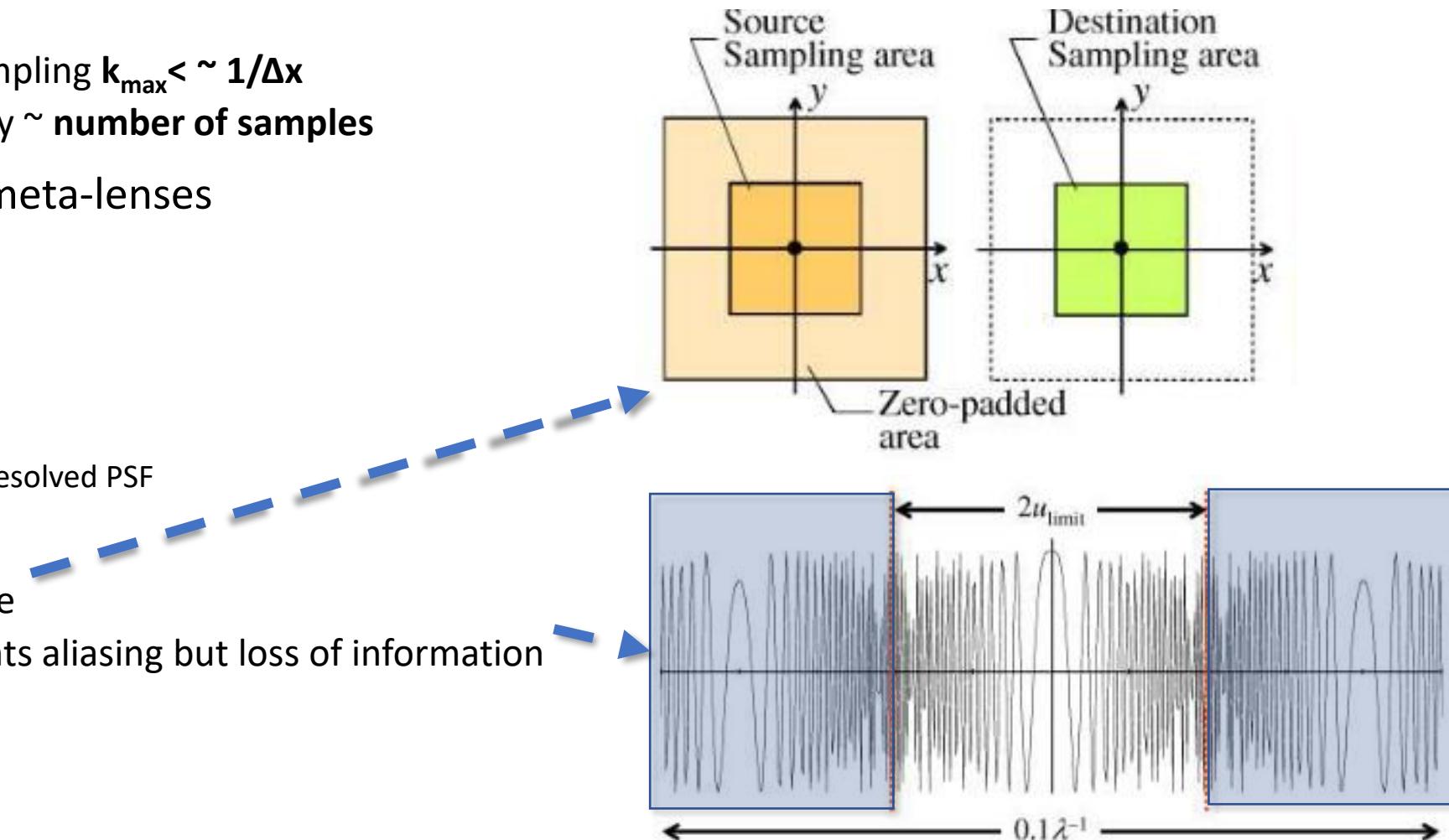
➢ 1 FFT

$$E(x, y, z) = \frac{\exp(ikz)}{i\lambda z} \frac{\exp(ik\frac{x^2 + y^2}{2z})}{F \left\{ E_0(u, v, 0) \exp \left[\frac{i\pi}{\lambda z} (u^2 + v^2) \right] \right\}}$$



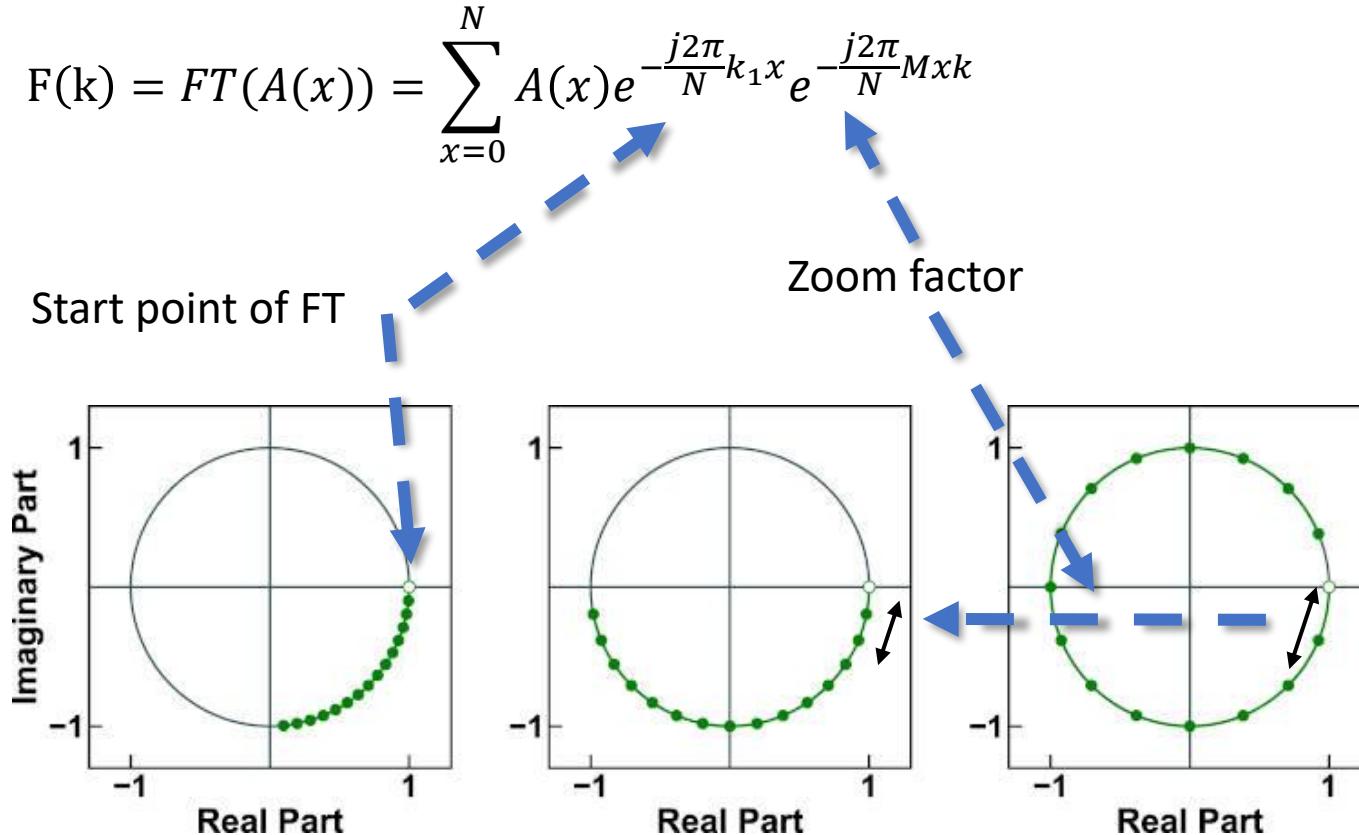
Numerical effects

- ❖ Errors arising from **discretization/numerical calculations**
- ❖ Standard FFT via Cooley-Tukey algorithm:
 - Aliasing effects
 - Angular width determined by sampling $k_{\max} < \sim 1/\Delta x$
 - **Angular resolution** determined by \sim **number of samples**
- ❖ Implications for high NA and meta-lenses
 - Sampling $\approx \lambda/n$
 - Diffraction limited spot profile
$$FWHM = \frac{\lambda}{2NA}$$
 - For $n=1.5$ and $NA = 0.75$
1 data point within FWHM -> poorly resolved PSF
- ❖ Possible improvements:
 - Zero padding -> memory intensive
 - Low pass filter (BLASM) -> prevents aliasing but loss of information
 - **Chirped Z-transform (zoom FFT)**



CZT implementation of DFT

Discrete Fourier transform in zoom band



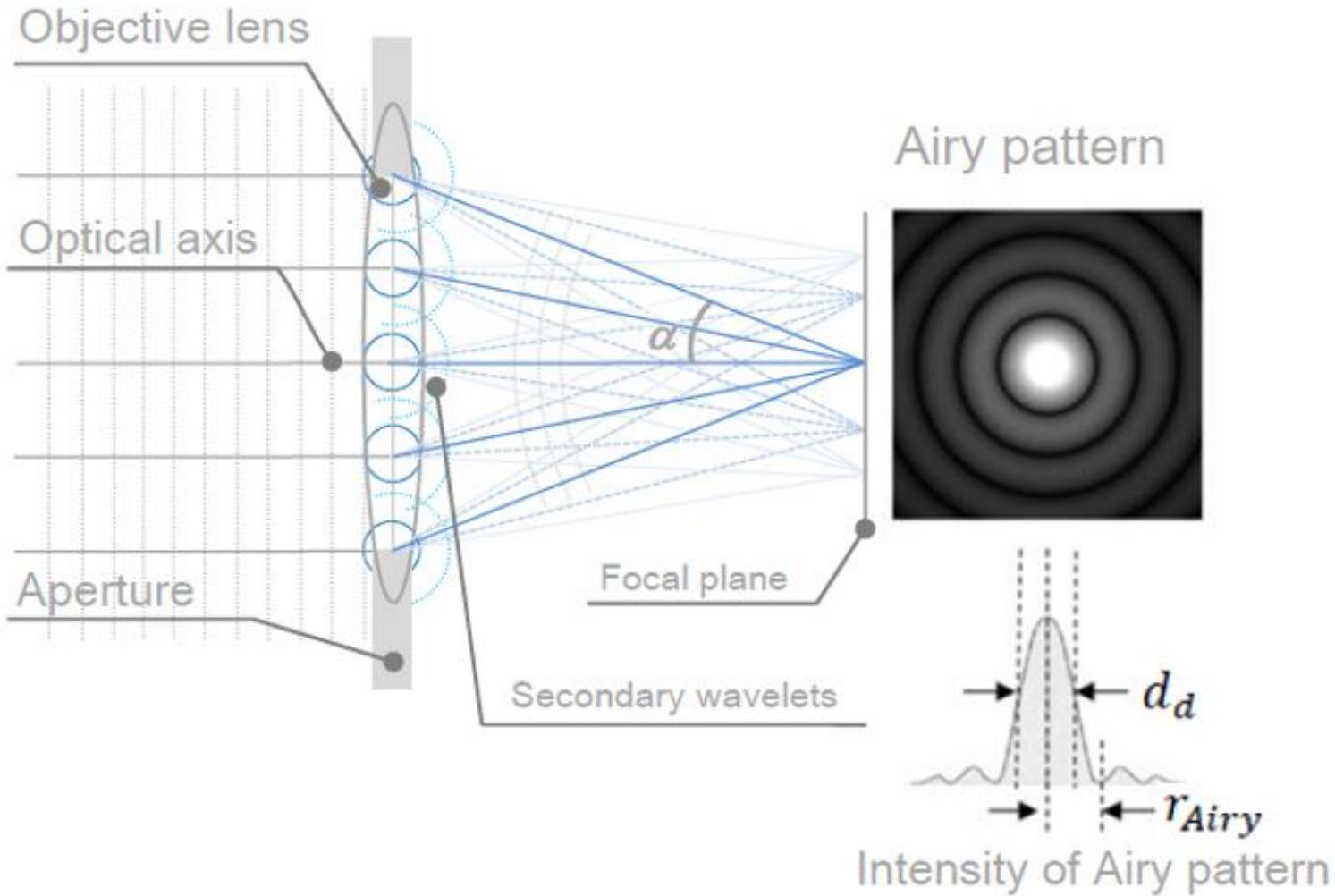
- ❖ Chirped Z transform (BlueStein)
 - DFT with control over sampling
 - Slower in computation (1 FFT → 2 FFT+IFFT)
- ❖ Trade off between stepsize and width of transform
 - Number of samples constant

Rewritten as convolution of “standard” FFT

$$F(k) = e^{-\frac{j\pi M}{N}k^2} IFT(FT\left(A(x)e^{-\frac{j2\pi}{N}k_1 x} e^{-\frac{j\pi}{N}x^2}\right) FT\left(e^{\frac{j\pi M}{N}(k-x)^2}\right))$$

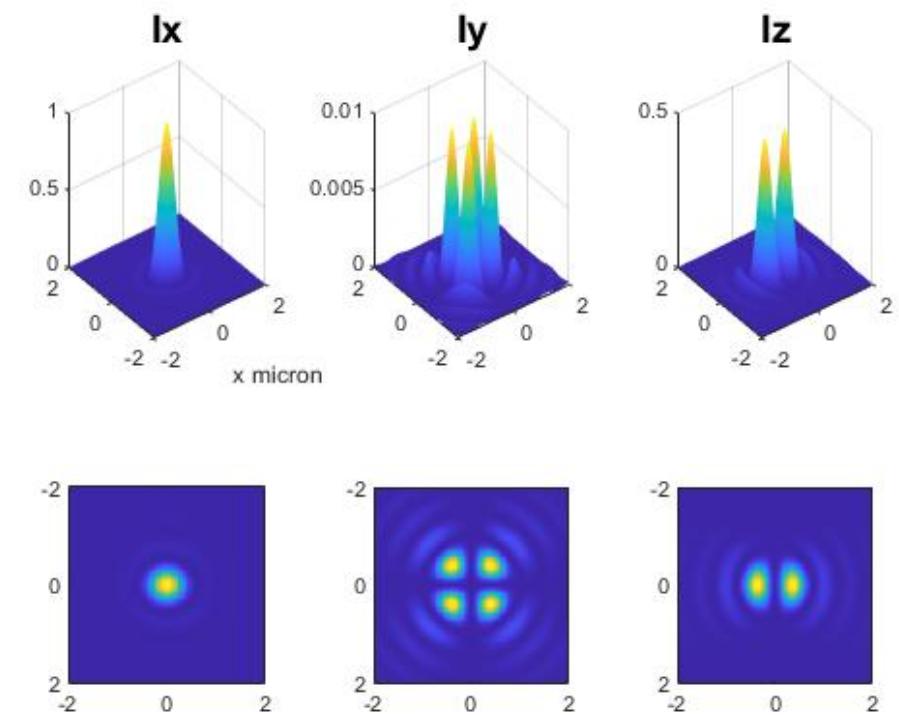
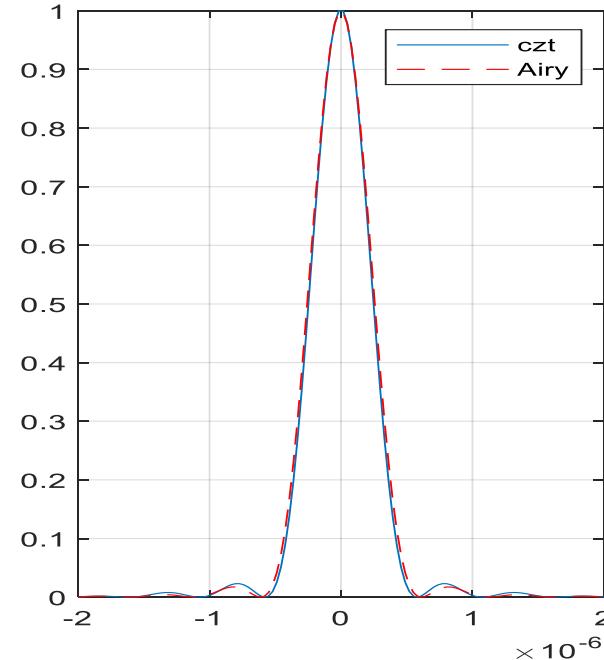
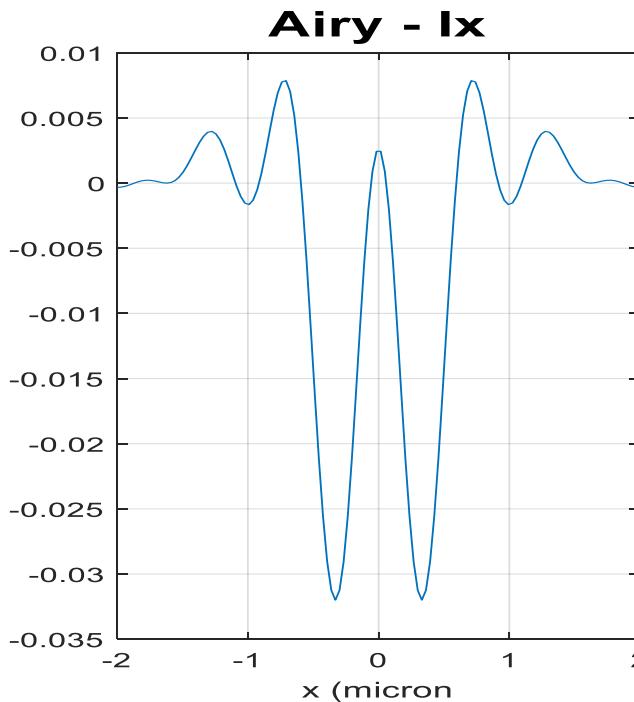
Numerical evaluation

- ❖ Reference case:
 - Lenses up to NA =1,2
 - Immersion or air based
- ❖ Parameters:
 - $\Lambda = 800\text{nm}$
 - $N = 200$ (original samples)
- ❖ Results compared to analytical solution:
$$I(x) = \left| \frac{2J_1(x)}{x} \right|^2$$
- ❖ Numerical cases:
 - Brute force
 - Angular spectrum / Rayleigh-Sommerfeld:
 - FFT
 - CZT
 - Paraxial/ Fresnel integral



Vectorial calculation

- ❖ **Brute force** integration: trapezium
- ❖ Most rigorous numerical approach
- ❖ For strong focusing a z-component of the field is produced
- ❖ Remaining error due to trapezoidal integration:
 - Improved by higher sampling
 - Other integration methods



Scalar calculation

❖ Angular spectrum method: non-paraxial

- No assumption on angles

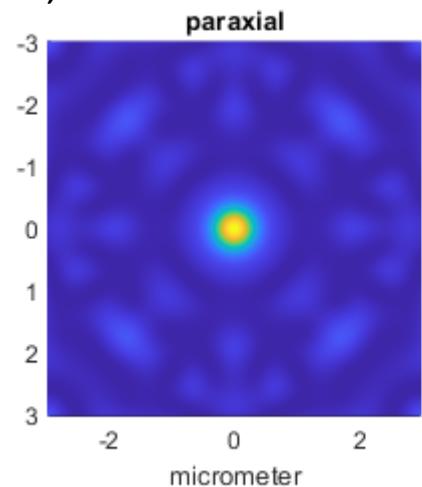
$$E(x, y, z) = \iint A(k_x, k_y; z=0) \exp\left[-i(k_x x + k_y y + k_z z)\right] dk_x dk_y$$

❖ Fresnel integral: paraxial

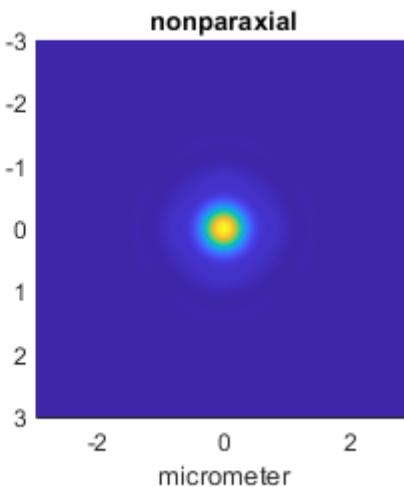
- Assumptions only valid for shallow angles

$$E(x, y, z) = \frac{\exp(ikz)}{i\lambda z} \frac{\exp(ik\frac{x^2 + y^2}{2z})}{2z} F \left\{ E_0(u, v, 0) \exp\left[\frac{i\pi}{\lambda z}(u^2 + v^2)\right] \right\}$$

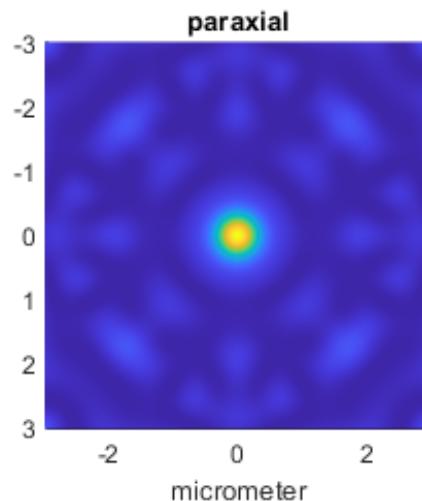
NA=0,61



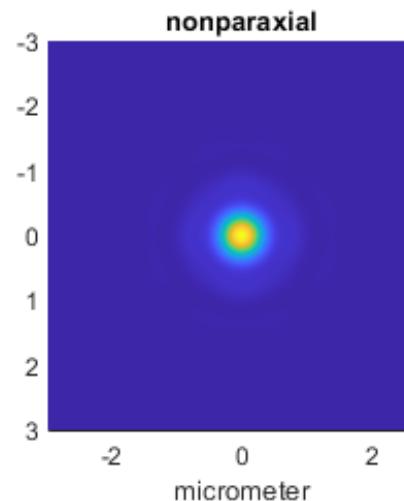
nonparaxial



NA=0,9

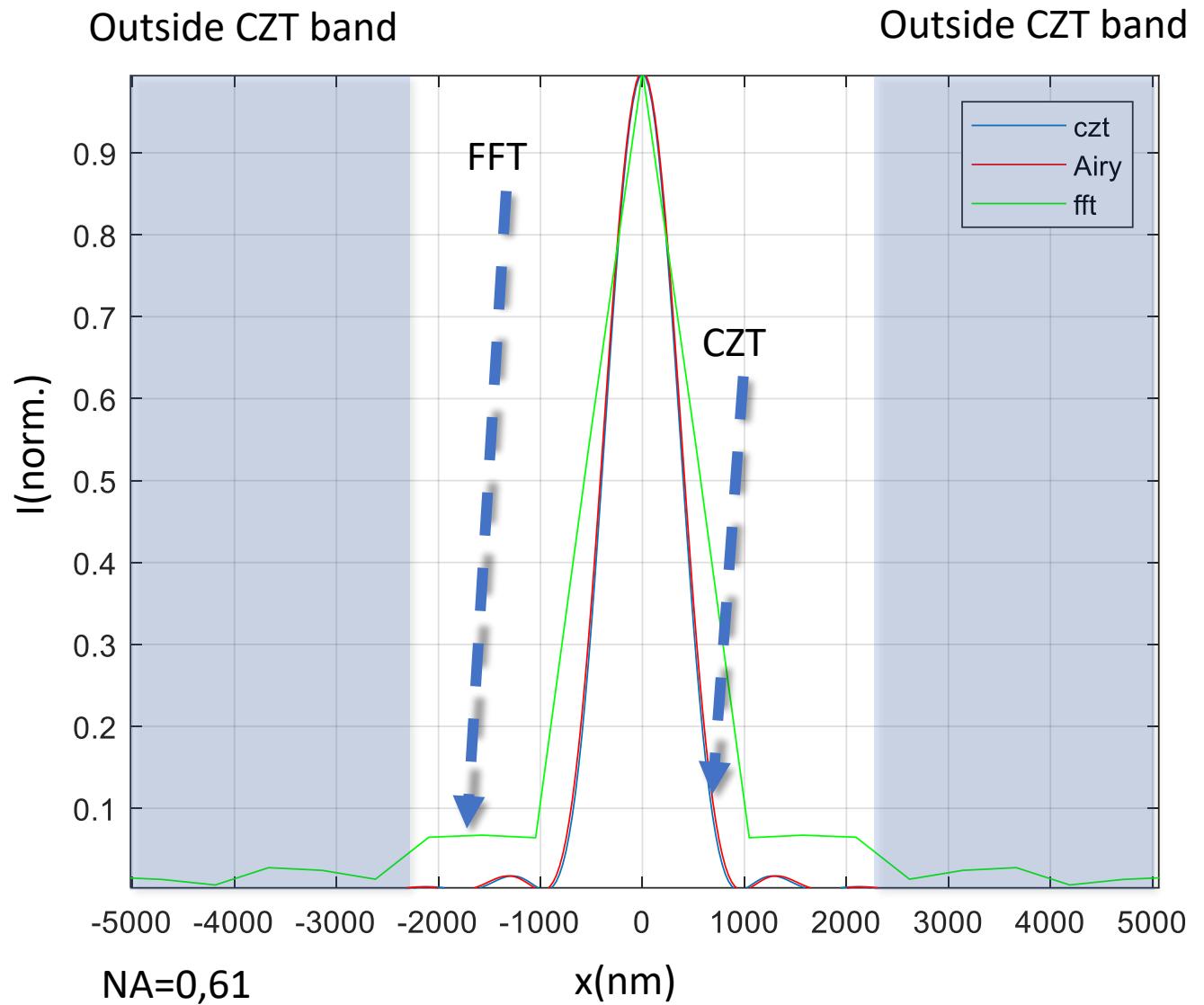


nonparaxial



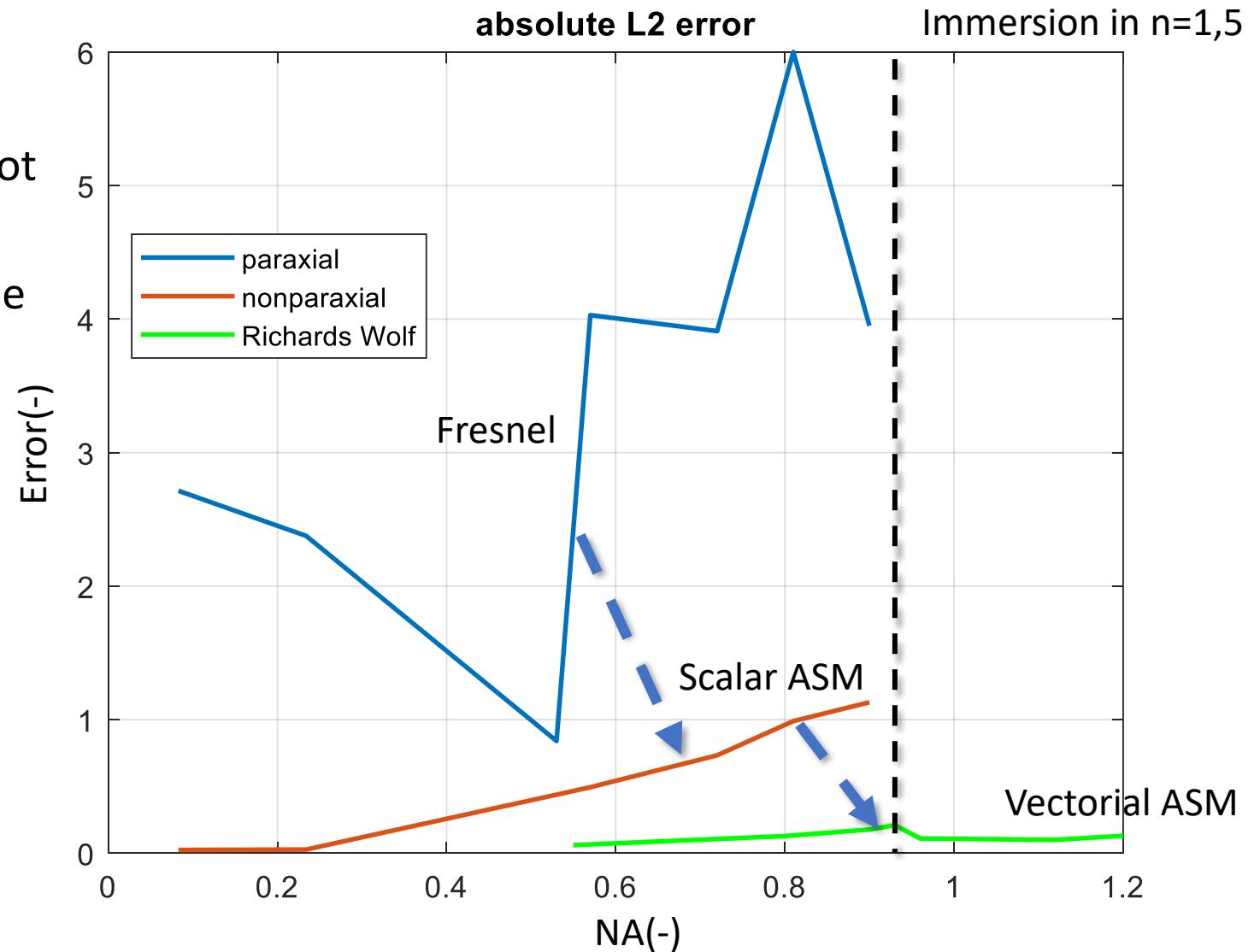
Zoom

- ❖ Standard DFT vs. CZT
- ❖ DFT
 - Stepsize determined by original
- ❖ CZT:
 - Well resolved airy disk
 - Smaller range
- ❖ Tested up to:
 - $\Delta x = \lambda/50$
 - Zoom x300



Error comparison

- ❖ Error vs. Analytical solution
- ❖ $L2 = |I_{num}(x) - I_{analytic}(x)|$
- ❖ CZT was applied to all cases to resolve spot
- ❖ As NA gets progressively higher more computational intense algorithms must be used:
 - Low NA: Fresnel
 - Intermediate NA: scalar ASM
 - High NA: vectorial ASM



Conclusions

- ❖ Standard diffraction integrals and FFT algorithms are not well suited for high NA/immersion lens calculations
- ❖ Accurate calculation of high NA focal spot requires:
 - Chirped Z-transform: improved image plane sampling
 - Vectorial angular spectrum calculations
- ❖ The computational cost rises as below

NA range	Method	Standard resolution	Zoom
<0,2	Fresnel	1 FFT	3 FFT
0,2 – 0,6	ASM + CZT	2 FFT	6 FFT
>0,6	Vectorial ASM	6 FFT	18 FFT

Reach us here!

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